Heat and Mass Transfer Between an Evaporative Interface in a Porous Medium and an External Gas Stream

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Steady state heat and mass transfer between a submerged evaporative interface in a porous medium and an external gas stream were analyzed theoretically. Local and average evaporation rates for laminar and turbulent flow over a flat plate as well as for fully developed laminar or turbulent flow near the thermal entry region of a duct were obtained as a function of the position of the evaporative interface. Calculated local evaporation rates are significantly lower than values computed by Luikov, who did not take into account the variation of evaporative surface temperature with recession, indicated both by the present analysis and by observations related to the drying of thick porous materials. Application to the analysis of the drying process is discussed.

This investigation is concerned with heat and mass transfer between an evaporating liquid in a porous medium and an external gas stream, an area of research which is of particular relevance to the field of drying. An analysis is presented of steady state heat and mass transfer at low mass transfer rates between a submerged evaporative interface parallel to the surface of a porous medium and an external gas stream. The results of the analysis permit calculation of both local and average evaporation rates for different geometries and flow regimes, provided that the location of the evaporative interface is known.

The use of a receding plane of evaporation to analyze drying processes was suggested by Gilliland in 1938 (*8*). Nissan, George, and Bolles (*19*) used the concept of a submerging plane of evaporation in order to develop a complete description of drying but their analysis neglects to take into account the influence of the fluid boundary layer. The direct precursor to this paper is a 1962 study by Luikov (*15*, *16*), who applied boundary-layer theory to the problem of evaporation of liquid from a porous body into a flowing air stream. Based on earlier experimental results (*17*), Luikov as well as Nissan had concluded that most of the evaporation during the drying process takes place in a very narrow layer at the junction between wet and dry regions. Accordingly, Luikov's analysis was based upon the hypothesis that all evaporation takes place from a submerged surface which is parallel to the surface of the porous medium. In contrast to Nissan's analysis (*19*), the effects of the fluid boundary layer as well as those of the porous medium on the evaporation process were taken into account.

There are two main distinctions between Luikov's previous analysis and the analysis reported in this paper. Whereas Luikov's solution depends upon the assumption that the velocity is constant across the fluid boundary layer, our analysis employs an integral representation that can be used for the more general flow situation in which the heat and mass transfer coefficients past a single step in wall temperature and concentration at $x = \xi$ are of the form

$$\frac{h(x,\xi)}{h_T} = \frac{k(x,\xi)}{k_T} = [1 - (\xi/x)^{\delta}]^{-\alpha} \tag{1}$$

Equation (1) is applicable to boundary-layer wedge flows at low mass transfer rates. Some specific cases considered in this paper are laminar and turbulent flow over a flat plate and fully developed laminar and turbulent flow near the thermal entrance region within a duct or channel of arbitrary geometry in which the wall shear stress is known. In general, the boundary condition imposed by the submerged surface of evaporation is such that the problem is analogous to the problem of boundary-layer mass transfer with first-order chemical surface reaction.

In his analysis, Luikov assumed that the temperature of the submerged evaporative surface regardless of its position within the porous medium was everywhere equal to the wet-bulb temperature based upon the ratio of heat to mass transfer coefficients for the fluid boundary layer. However, for most materials this ratio will be considerably less than the ratio of heat to mass transfer coefficients for the porous medium. As a result, the temperature of the evaporative surface will increase as the distance between the evaporative surface and the surface of the porous medium increases. This temperature increase provides a logical physical explanation for the temperature rise from the wet bulb to the pseudo wet-bulb temperature observed during drying of thick porous materials by Nissan and co-workers (*19*), Adams (*1*), and Luikov (*17*). In this paper an approximate analysis is presented which takes this temperature rise into account.

FORMULATION OF THE PROBLEM AND METHOD OF SOLUTION

Consider the situation shown in Figure 1 in which a stream of gas of upstream temperature t_∞ passes over the surface of a wet porous material. An analysis of the heat transfer process is to be made on the basis of the following conditions and assumptions:

1. At any time, liquid is evaporating at a surface parallel to and situated a distance τ below the surface of the porous material.
2. Heat transfer within the porous medium between $y = 0$ and $y = -\tau$ is represented as steady one-dimensional

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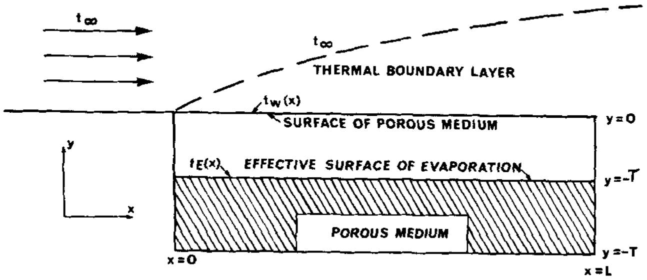

Fig. 1. Evaporation from effective distance τ below body surface.

conduction in the y direction characterized by a constant effective thermal conductivity k_{eff}. All other heat transfer within the medium and between the medium and surroundings is negligible.

3. The effect of surface roughness on heat transfer is negligible.
4. The effect of normal surface velocity (mass transfer) on heat transfer is negligible.
5. Fluid properties in the boundary layer are constant.

These conditions and assumptions reduce the problem to one of boundary-layer heat transfer from a surface of varying wall temperature $t_w(x)$, where the subscript w denotes the physical boundary of the porous medium. The wall temperature may be related to the wall heat flux $q_w(x)$, and thus the rate of evaporation, by the following equation derived by Klein and Tribus (*24*) based upon superposition of step-wall temperature solutions:

$$t_w(x) - t_\infty = \frac{\delta}{\Gamma(1-\alpha)\Gamma(\alpha)} \int_{\xi=0}^{\xi=x} \frac{[1-(\xi/x)^{\delta}]^{\alpha-1}}{xh_T}\, q_w(\xi)\, d\xi \tag{2}$$

where h_T is defined as

$$h_T = c_T \frac{k}{x} (N_{Re_x})^m (N_{Pr})^p \tag{3}$$

The values of the constants c_T, m, α, p, and δ for several cases of interest are summarized in Table 1. Equation (3) does not contain the dimensionless Gukhman number, which has been reported to be of significance in liquid evaporation processes (*15*).

From assumption 2 and the requirement of continuity of heat flux at the surface, the wall heat flux $q_w(x)$ is related to the wall temperature $t_w(x)$ by the equation

$$q_w(x) = -k_{\text{eff}} \left.\frac{\partial t}{\partial y}\right|_{y=0} = \frac{k_{\text{eff}}}{\tau}[t_E(x) - t_w(x)] \tag{4}$$

where k_{eff} is the effective thermal conductivity of the porous medium between $y = 0$ and $y = -\tau$. Eliminating $t_w(x)$ between Equations (2) and (4) and rearranging, we can obtain a Volterra integral equation of the second kind in $q_w(x)$:

$$q_w(x) = [t_E(x) - t_\infty]\frac{k_{\text{eff}}}{\tau} + \lambda_1 \int_{\xi=0}^{\xi=x} K_1(x,\xi) q_w(\xi) d\xi \tag{5}$$

where the constant λ_1 and the unsymmetrical kernel $K_1(x, \xi)$ are given by the expressions

$$\lambda_1 = -\frac{\delta(k_{\text{eff}}/\tau)}{\Gamma(1-\alpha)\Gamma(\alpha)} \tag{6}$$

$$K_1(x,\xi) = \frac{[1-(\xi/x)^{\delta}]^{\alpha-1}}{xh_T} \tag{7}$$

To solve Equation (5) for $q_w(x)$, the axial dependence of the temperature of the evaporative surface $t_E(x)$ must be known. It is therefore necessary to consider the mass transfer aspects of the problem. The mass transfer equation analogous to Equation (5) is

$$J_w(x) = [X_E(x) - X_\infty]\frac{cD_{\text{eff}}}{\tau} + \lambda_2 \int_{\xi=0}^{\xi=x} K_2(x,\xi) J_w(\xi) d\xi \tag{8}$$

where

$$\lambda_2 = \lambda_1 \frac{cD_{\text{eff}}}{k_{\text{eff}}} \tag{9}$$

and

$$K_2(x,\xi) = K_1(x,\xi)\frac{h_T}{k_T} \tag{10}$$

The heat and mass fluxes are related by a local energy balance at the evaporative surface:

$$\frac{q_w(x)}{J_w(x)} = -\Delta H \tag{11}$$

Equations (8) and (11) are valid for low mass transfer rates and for $X_E(x) << 1.0$.

At the surface of evaporation, the concentration of evaporating component in the vapor and the temperature are assumed to be related by the equilibrium vapor pressure-temperature relationship for the evaporating liquid. This relationship along with Equations (5), (8), and (11) completely describes the problem as formulated. However, in order to obtain an analytical solution, it is necessary to linearize the vapor pressure-temperature equation:

$$X_E(x) = A\ t_E(x) + B \tag{12}$$

Elimination of $J_w(x)$ and $X_E(x)$ from Equation (8) by using Equations (11)and (12) and comparison of Equations (5) and (8) reveal that $t_E(x)$ and $q_W(x)$ are linearly related. If the constants A and B of Equation (12) are then eliminated in favor of appropriate temperatures and concentrations by using the linearized vapor pressure-

Table 1. Kernels for Step-Wall Temperature Superposition

$$K_1 = \frac{[1-(\xi/x)^{\delta}]^{\alpha-1}}{xh_T}$$

$$h_T = c_T(k/x)(N_{Re_x})^m(N_{Pr})^p$$

System‡	c_T	δ	α	m†	p
1. Laminar flow over flat plate, $N_{Pr} << 1$	0.564	1	1/2	1/2	1/2
2. Laminar flow over flat plate, $N_{Pr} > 1$	0.332	3/4	1/3	1/2	1/3
3. Turbulent flow over flat plate, $N_{Pr} > 1$	0.029	9/10	1/9	4/5	0.49
4. Developed laminar or turbulent flow in a duct or channel of arbitrary geometry close to the step in wall temperature	0.538	1	1/3	2/3	1/3

† For case 4, the Reynolds number is based upon the friction velocity $v^* = \sqrt{\tau_w/\rho}$, where τ_w = wall shear stress. For all other cases, N_{Re_x} is based upon the free-stream velocity U_∞.

‡ The parameters for cases 1 to 3 have been tabulated by Hanna (*9*), who developed a step-wall, heat-flux superposition technique, which also could have been employed in the present analysis. The parameters for case 4 were obtained by using Lighthill's solution (*14*) and an arbitrary wall shear stress. For region of applicability in turbulent flow, see references 3 and 7.

temperature equation, a compact and convenient form of the relation between $q_w(x)$ and $t_w(x)$ may be obtained:

$$t_E(x) = \left[\frac{t_E(0) - t_{AS}}{t_E(0) - t_\infty}\right] \frac{\tau}{k_{\text{eff}}} q_w(x) + t_{AS} \quad (13)$$

The constant $t_E(0)$ is the temperature of the evaporative surface at the upstream edge of the porous body, that is at $x = 0$. Because the thermal resistance of the fluid boundary layer vanishes at this location, $t_E(0)$ is independent of the position of the evaporative surface and depends upon the ratio of heat to mass transfer coefficients for the porous medium in accordance with the energy balance:

$$\frac{\{X_E(0) - X_\infty\}\ \Delta H}{\{t_\infty - t_E(0)\}\ \{1 - X_E(0)\}} = \frac{k_{\text{eff}}}{cD_{\text{eff}}} \quad (14)$$

The constant t_{AS} is the temperature at the evaporative surface when evaporation takes place at the surface of the porous body. It may be calculated from the surface energy balance:

$$\frac{(X_{AS} - X_\infty)\ \Delta H}{(t_\infty - t_{AS})\ (1 - X_{AS})} = \frac{h_T}{k_T} \quad (15)$$

where the ratio of heat to mass transfer coefficients h_T/k_T for the fluid boundary layer can be obtained from the expression

$$\frac{h_T}{k_T} = \tilde{c}_p\ (N_{Sc}/N_{Pr})^{(1-p)} \quad (16)$$

The quantity t_{AS} will be either equal to or very nearly equal to the psychrometric wet-bulb temperature, depending upon the value of the exponent $1 - p$ in Equation (16) for the case in question (see Table 1).

Elimination of $t_E(x)$ from Equation (5) by using Equation (13) linearizes the Volterra integral equation in $q_w(x)$:

$$q_w(x) = q_w(0) + P\lambda_1 \int_{\xi=0}^{\xi=x} K_1(x, \xi) q_w(\xi) d\xi \quad (17)$$

where $q_w(0)$ is the heat flux at the leading edge and is given by

$$q_w(0) = (t_E(0) - t_\infty) \frac{k_{\text{eff}}}{\tau} \quad (18)$$

while P is a parameter related to the temperature of the submerged evaporative surface and is expressed as

$$P = \frac{t_E(0) - t_\infty}{t_{AS} - t_\infty} \quad (19)$$

Equation (17) has been solved by expanding $q_w(x)$ in a MacLaurin series in the variable Pz:

$$q_w(x) = q_w(0) \left[1 + \sum_{n=1}^{\infty} (-1)^n d_n (Pz)^n\right] \quad (20)$$

where

$$z = \frac{k_{\text{eff}}}{\tau h_T} \quad (21)$$

is the ratio of the thermal resistance of the fluid boundary layer to the thermal resistance of the porous body. Since h_T is a function of x, as can be seen from Equation (3), the relative resistance parameter z increases with distance downstream from the leading edge according to the relation:

$$z = (\text{constant}) x^{1-m} \quad (22)$$

Note that for any given value of x, z decreases as the surface of evaporation recedes, that is, as τ increases.

The coefficients d_n are determined by substituting Equation (20) into the right-hand side of Equation (17), performing the integration, and then comparing coefficients, with the result

$$d_n = \sum_{i=1}^{n} D_i \quad (23)$$

$$D_n = \frac{\Gamma\left[\frac{n - (n-1)m}{\delta}\right]}{\Gamma(1-\alpha)\Gamma\left[\alpha + \frac{n-(n-1)m}{\delta}\right]} \quad (24)$$

The average surface heat flux, which is directly related to the rate of evaporation by Equation (11), may be obtained by integrating Equation (20) over the length of the bed:

$$q_w(Av) = \int_0^L \frac{q_w(x)dx}{L} \quad (25)$$

The result is

$$q_w(Av) = q_w(0) \left[1 + \sum_{n=1}^{\infty} (-1)^n \frac{d_n\ (Pz_L)^n}{1 + n(1-m)}\right] \quad (26)$$

where

$$z_L = \frac{k_{\text{eff}}}{\tau h_{TL}} \quad (27)$$

and h_{TL} is the constant wall temperature coefficient evaluated at $x = L$.

Although convergent for all values of Pz and Pz_L, Equations (20) and (26) can only be used over a limited range due to increasing round-off error with increasing z. However, for large values of Pz, Equation (17) may be solved by expanding $q_w(x)$ in an asymptotic power series $1/Pz$ with the result:†

$$q_w(x) = q_w(0) \sum_{n=1}^{\infty} (-1)^{n-1} c_n (1/Pz)^n \qquad \text{(for large } Pz) \quad (28)$$

where

$$c_n = \prod_{i=1}^{n} C_i \quad (29)$$

and

$$C_n = \frac{\Gamma(1-\alpha)\Gamma\left[\alpha + \frac{1-n(1-m)}{\delta}\right]}{\Gamma\left[\frac{1-n(1-m)}{\delta}\right]} \quad (30)$$

Similarly, for large values of Pz, the average surface heat flux can be obtained by separating the integral of Equation (25) into two parts:

$$q_w(Av) = \int_0^{x_A} \frac{q_w(x)dx}{L} + \int_{x_A}^{x} \frac{q_w(x)dx}{L} \quad (31)$$

where the constant x_A is sufficiently large such that the asymptotic series, Equation (28), may be used in integrating the right-hand term. Note that as $z \to \infty$ corresponding to evaporation very close to the surface of

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† For cases 2 and 3, the remainder of the series expression of Equation (28) for $n > [(1/\alpha) - 1]$ is indeterminate. For the laminar flat plate case, case 2, a more useful asymptotic expression is:

$$q_w(x) = \sum_{n=0}^{\infty} (-1)^n c_n' (1/Pz)^{[1 + (3/2)n]} \quad (28a)$$

where

$$c_n' = \frac{\Gamma\left(1 + \frac{3}{2}n\right)\left[\Gamma\left(\frac{5}{2}\right)\right]^{\left(\frac{3}{2}\right)n}}{n!} \quad (28b)$$

Equation (28*a*) is presented in somewhat different form by Levich (*13*), who considered the analogous problem of laminar boundary-layer mass transfer from a flat plate with first-order chemical surface reaction.

the porous body, Equation (28) reduces to

$$\underset{Z\to\infty}{q_w(x)} = \frac{q_w(0)}{Pz} = h_T(t_{AS} - t_\infty) \qquad (32)$$

From Equation (31) the limiting value of $q_w(Av)$ as $z \to \infty$ is

$$\underset{Z\to\infty}{q_w(Av)} = \frac{q_w(0)}{mPz_L} = \frac{h_{TL}}{m}(t_{AS} - t_\infty) \qquad (33)$$

Equations (32) and (33) exhibit the correct physical dependence of the heat flux on the fluid boundary layer for the limiting case of no recession of the evaporative surface.

CALCULATED RESULTS

It is convenient to present the results of the mathematical analysis of the previous section in terms of dimensionless quantities.

The local f function is defined as the ratio of the local heat flux to the heat flux at the leading edge:

$$f(Pz) = q_w(x)/q_w(0) \qquad (34)$$

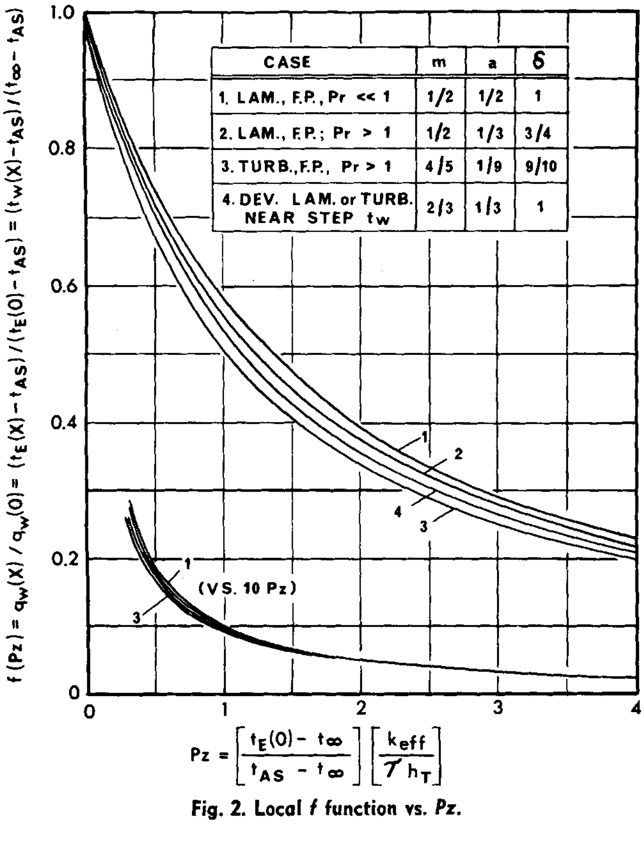
The function $f(Pz)$ is directly related to the local temperature of the evaporative surface $t_E(x)$ and the local temperature of the surface of the porous body $t_w(x)$. From Equations (4), (13), (18), and (34)

$$f(Pz) = \frac{t_E(x) - t_{AS}}{t_E(0) - t_{AS}} = \frac{t_w(x) - t_{AS}}{t_\infty - t_{AS}} \qquad (35)$$

The local f function may be calculated by using Equations (20) and (28).

The average $\bar{f}$ function is defined as the ratio of the heat flux averaged over the length of the porous medium to the heat flux at the leading edge:

$$\bar{f}(Pz_L) = q_w(Av)/q_w(0) \qquad (36)$$



CASE	m	a	δ
1. LAM., F.P., Pr << 1	1/2	1/2	1
2. LAM., F.P.; Pr > 1	1/2	1/3	3/4
3. TURB., F.P., Pr > 1	4/5	1/9	9/10
4. DEV. LAM. or TURB. NEAR STEP tw	2/3	1/3	1

Fig. 2. Local f function vs. Pz.

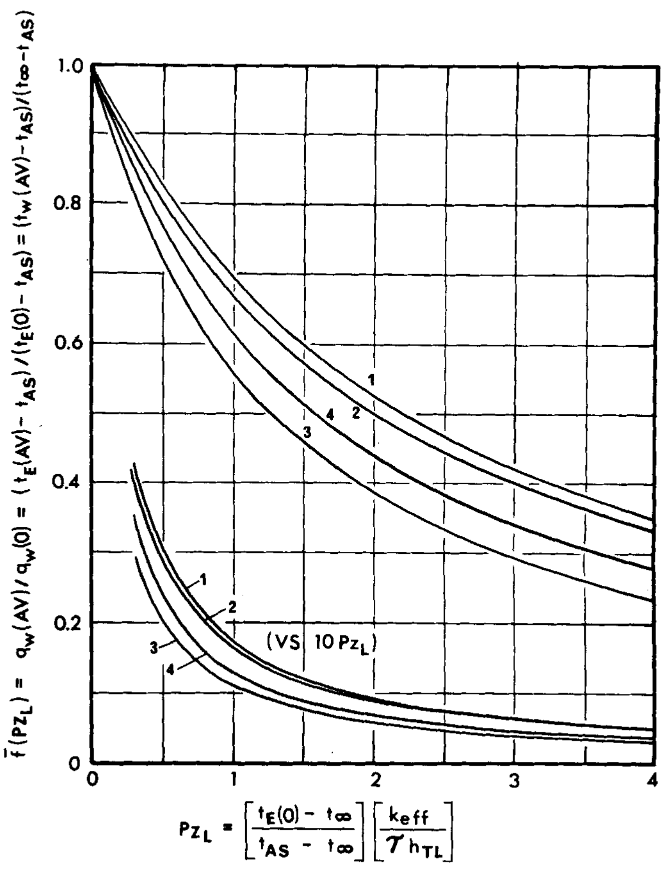


Fig. 3. Average $\bar{f}$ function vs. Pz_L.

The $\bar{f}$ function is related to the average temperature of the evaporative surface $t_E(Av)$ and of the surface of the porous body $t_w(Av)$ by the equations:

$$\bar{f}(Pz_L) = \frac{t_E(Av) - t_{AS}}{t_E(0) - t_{AS}} = \frac{t_w(Av) - t_{AS}}{t_\infty - t_{AS}} \qquad (37)$$

and may be calculated by using Equations (28) and (31).

The average $\bar{g}$ function is defined as the average heat flux relative to the average heat flux for evaporation from the surface of the porous body:

$$\bar{g}(Pz_L) = q_w(Av)/[q_w(Av)]_{Z\to\infty} \qquad (38)$$

The $\bar{g}$ function may be calculated from Equation (39), which is obtained Equations (33), (36), and (38):

$$\bar{g}(Pz_L) = mPz_L\,\bar{f}\,(Pz_L) \qquad (39)$$

Curves of $f(Pz)$ as a function of Pz and $\bar{f}(Pz_L)$ and $\bar{g}(Pz_L)$ as functions of Pz_L are shown in Figures 2, 3, and 4 for four cases of interest, namely:

1. Laminar flow over a flat plate, $N_{Pr} << 1$. The f function for this case reduces to the analytical expression

$$f(Pz) = \exp\frac{(Pz)^2}{\sqrt{\pi}}\,\text{erfc}\,\frac{Pz}{\sqrt{\pi}} \qquad (40)$$

Luikov (*15*) obtained a result which is similar in form to Equation (40) by direct integration of the boundary-layer energy equation, subject to the approximation that the longitudinal velocity component across the thermal boundary layer is constant at $u_{th} = (5/8)U_\infty$. However, Luikov's result is rigorously correct only for very low Prandtl numbers. Furthermore, Luikov's result does not include the parameter P, which takes into account the temperature rise of the submerging evaporative surface as it recedes into the porous medium. This temperature rise can have a

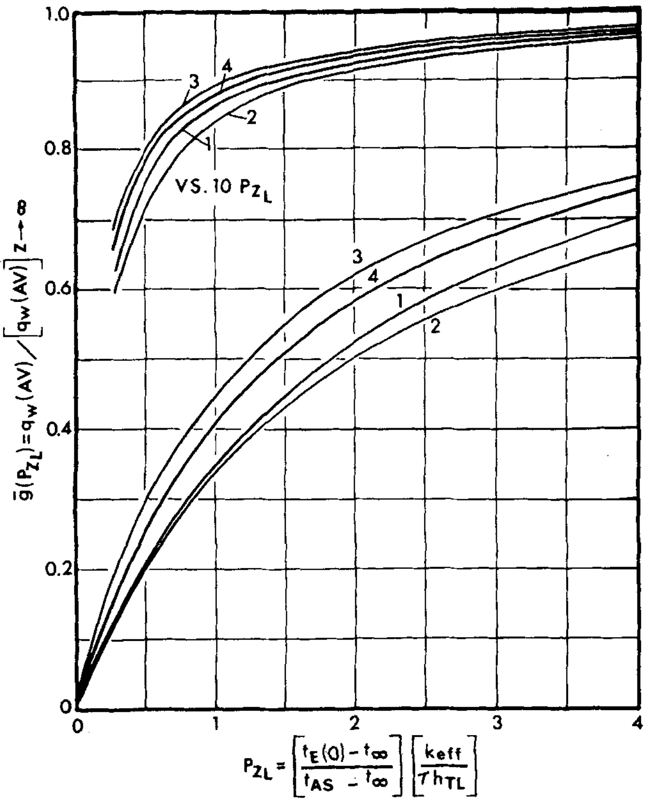


Fig. 4. Average $\bar{g}$ function vs. Pz_L.

significant effect upon the heat flux, as will be discussed subsequently.

2. Laminar flow over a flat plate, $N_{Pr} > 1$. The curve for the local heat flux function $f(Pz)$ for this case is identical with a curve presented by Chambre and Acrivos (*6*), who considered the analogous problem of laminar boundary-layer mass transfer from a flat plate at which a first-order chemical surface reaction is occurring.

3. Turbulent flow over a flat plate, $N_{Pr} > 1$. The first five terms of the series expression for the f function for this case have been presented in an article by Rosner (*21*) on the influence of the turbulent boundary layer upon the kinetics of surface reactions.

4. Developed laminar or turbulent flow in a duct or channel of arbitrary geometry close to thermal entrance region. The constant wall heat transfer coefficient for this case was obtained by using Lighthill's solution (*14*) and an arbitrary constant wall shear stress. For turbulent flow, the extent of the region downstream from the leading thermal edge over which this treatment applies rigorously is usually very small, as can be deduced from the work of Gardner and Kestin (*7*). A more useful relationship for turbulent duct flow has been presented recently by Back and Seban (*3*) for which the f, $\bar{f}$, and $\bar{g}$ functions correspond approximately to the results for case 3, turbulent flow over a flat plate.

Asymptotic formulas useful for highly accurate calculation of f and $\bar{f}$ for small and large values of their respective arguments are presented in reference *18*. Note that for Pz and Pz_L greater than 40, f and $\bar{f}$ are given approximately by $1/(Pz)$ and $1/(mPz_L)$, respectively.

DISCUSSION AND APPLICATION OF RESULTS

Comparison with Luikov's Work

Local evaporation rates predicted by the present analysis and by Luikov (*15*) were compared for evaporation of water from a porous ceramic into a flowing air stream for the following specific conditions:

k_{eff} = 0.2 kcal./(hr.)(meter)(°C.) = 0.1344 B.t.u./(hr.)(ft.)(°F.)
$N_{Re_x} = 6 \times 10^4$
$N_{Pr} = 0.7$
x = 20 mm.

Luikov calculated values of a modified relative resistance parameter:

$$K = z/\sqrt{\tau} \qquad (41)$$

based upon the assumptions that the flow was laminar and that the velocity across the thermal boundary layer was constant at $u_{th} = (5/8)U_\infty$. For τ = 0.5 and 3 mm., he computed K = 1.5 and 0.25, respectively.

Luikov expressed evaporation rates in terms of dimensionless ratios of heat transfer coefficients. In particular, he defined a local heat transfer coefficient $h_p(x)$ in terms of the psychrometric temperature difference by the equation

$$h_p(x) = \frac{q_w(x)}{t_{AS} - t_\infty} \qquad (42)$$

and presented a curve of the ratio $h_p(x)/h_T$ vs. K from which values of $h_p(x)/h_T$ equal to 0.86 and 0.34 can be obtained corresponding to τ = 0.5 and 3 mm., respectively.

In terms of the current analysis, the ratio $h_p(x)/h_T$ can be shown to be identical with the ratio of the local heat flux for evaporation from a submerged surface to the heat flux for evaporation from the body surface:

$$\frac{h_p(x)}{h_T} = \frac{q_w(x)}{[q_w(x)]_{Z \to \infty}} = Pz\ f\ (Pz) \qquad (43)$$

Since P depends upon the temperature and humidity of the air stream, these quantities were arbitrarily chosen at t_∞ = 150°F. and X_∞ = 0.01 for comparison. Other required parameters were temperature insensitive and were based on an average temperature of 100°F. as:

ΔH = 18,500 B.t.u./lb.-mole
h_T/k_T = 6.5 B.t.u./(lb.-mole)(°F.) [from Equation (16)]
k_{eff}/cD_{eff} = 210 B.t.u./(lb.-mole)(°F.)

The effective diffusivity D_{eff} was estimated by using the Bruggeman equation (*3*):

$$D_{eff} = D_v \epsilon^{3/2} \qquad (44)$$

where the void fraction for the porous ceramic in question, ϵ, is 0.40 (*24*).

The temperatures $t_E(0)$ and t_{AS} were calculated from Equations (14) and (15) to be 133.4° and 79.4°F., respectively, and a value of P was calculated from Equation (19) equal to 0.234. Equation (41) was used to calculate z by using Luikov's K values. Curve 1 of Figure 2 was employed to obtain $f(Pz)$ in order to retain consistency for the comparison. The dimensionless ratios $h_p(x)/h_T$ corresponding to τ = 0.5 and 3 mm. were 0.435 and 0.0967. Thus, the evaporation rate based on the present analysis is roughly one-half the rate predicted by Luikov at τ = 0.5 mm. and less than one-third of Luikov's rate at τ = 3 mm.

The marked difference in evaporation rates between the present analysis and Luikov's analysis is because the latter is based on a larger temperature-difference driving force between evaporative surface and air stream as a result of the assumption that the temperature at the evaporative surface is everywhere equal to the wet-bulb temperature for all values of τ. The relation between the dimensionless temperature of the evaporative surface as pre-

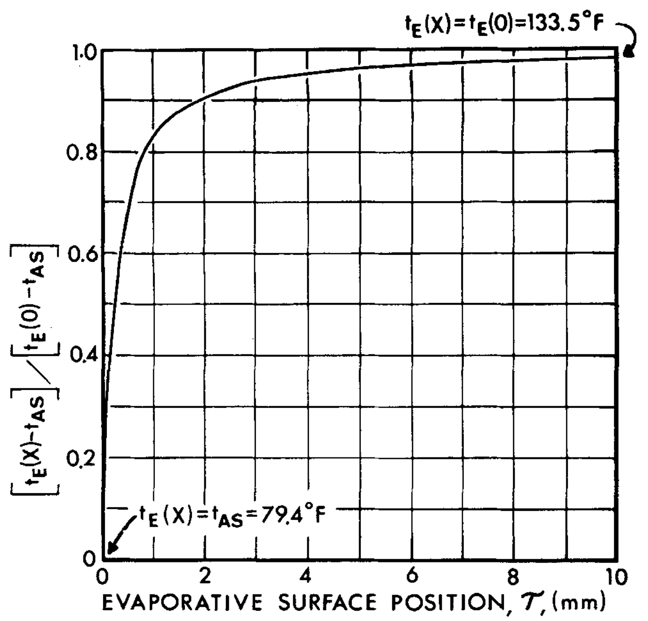


Fig. 5. Local dimensionless temperature vs. position of evaporative surface in a porous ceramic. t_∞ = 150°F.

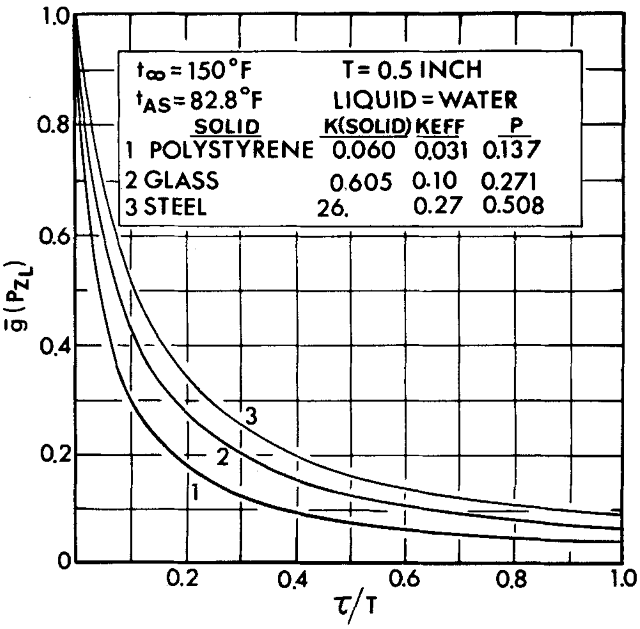


Fig. 6. Effect of thermal conductivity of the porous medium on the normalized evaporation rate.

dicted by the present analysis and the position of the evaporative surface is shown in Figure 5. Note that a relatively small recession of the plane of evaporation from the body surface can cause a sharp increase in the temperature of the evaporative surface and a corresponding decrease in heat flux.

Another important feature is the fact that $t_E(x)$ approaches an asymptotic value of $t_E(0)$ (133.5°F. in this case) for large τ. This behavior is due to the fact that the transport process is largely controlled by the porous medium once the plane of evaporation recedes a sufficient distance. Note that if the boundary-layer resistance were entirely negligible, the temperature of the evaporative surface at any x for all values of τ greater than zero would be $t_E(0)$, the temperature at the leading edge. In relatively thick porous media in which no heat transfer takes place across the bottom surface, the theoretical analysis predicts the existence of a region within the porous body throughout which the temperature is approximately equal to $t_E(0)$. Nissan and co-workers (*19*) had previously observed this characteristic in connection with experimental drying studies and assigned $t_E(0)$ the name *pseudo wet-bulb temperature.*

Parametric Studies

The theoretical analysis may be used to assess in a detailed, quantitative manner, the effects of variables such as air velocity, humidity, and temperature, as well as the physical properties of the porous medium and the evaporating liquid on the rate of evaporation from a submerged interface (*18*). The procedure and some illustrative results will now be presented. As an example, the ratio of the average heat flux for a situation in which the surface of evaporation has receded (z, τ finite) to the average heat flux for evaporation from the surface of the porous body ($\tau = 0$, $z \to \infty$) is a useful quantity for the interpretation of drying experiments. If variations in latent heat of vaporization with temperature can be neglected, the $\bar{g}$ function is the average rate of evaporation normalized by the average rate of evaporation from a liquid surface. On the basis of quasi steady state behavior for each τ, the $\bar{g}$ function may be used to calculate the rate of evaporation throughout the entire drying process as a function of the position of the evaporative surface, τ.

In Figure 6 are compared normalized rates of evaporation of water from 0.5-in. deep beds of uniform, random, densely packed polystyrene, glass, and steel spheres under identical air stream conditions. Values of $\bar{g}$ (Pz_L) were obtained from curve 4 of Figure 2 corresponding to developed flow near the thermal entrance region of a duct. Values of K_{eff} were calculated by the method of Kunii and Smith (*12*), and a value of h_{TL} equal to 5.90 B.t.u./(hr.)(sq.ft.)(°F.) corresponding to a bed length of 3¾ in. was used. As might be expected, the normalized evaporation rates are arranged in the order of the thermal conductivities. Actually the net effect is due to two competing factors: z, which increases directly with k_{eff}; and P, which decreases with increasing k_{eff} according to a modification of (19) as a consequence of the linearization of the vapor pressure function:

$$P = \frac{A\Delta H + (k_T/h_T)}{A\Delta H + (k_{\text{eff}}/cD_{\text{eff}})} \tag{45}$$

with z exerting a more pronounced influence. The physical significance of lower values of P for higher conductivity media is that for a fixed evaporative interface position, the overall driving force $t_\infty - t_E(0)$ is larger for glass than for steel.

The absolute evaporation rate R_A may be obtained by multiplying the normalized evaporation rate $\bar{g}(Pz_L)$ by the saturated surface evaporation rate R_S:

$$R_A = R_S\bar{g}(Pz_L) = \frac{h_{TL}}{\Delta H'}(t_\infty - t_{AS})\bar{g}(Pz_L) \tag{46}$$

Further studies showing the predicted effects of liquid properties and external conditions such as air temperature, and velocity for specific cases are presented elsewhere (*18*).

APPLICATION TO THE ANALYSIS OF DRYING DATA

In applying the theoretical formulation to the analysis of drying data, it is important to take note of several major assumptions: the uniform recession of the evaporative interface, the steady state temperature distribution, the linear temperature distribution above the plane of evaporation, the neglect of axial conduction within the porous medium, the adiabatic evaporative interface, and the linearization of the vapor-pressure-temperature relationship. In addition there are a number of limitations which are not unduly restrictive in most cases: the constant property requirement, negligible normal velocity (mass injection), negligible interfacial resistance, exclusion of viscous dissipation, thermal diffusion and diffusion thermo effects, and the requirement that the porous medium is rigid and does not interfere with the evaporating entity through chemical reaction, chemisorption, or vapor-pressure lowering.

In relation to the assumption that the effective position of the evaporation plane is parallel to the surface, it is to be noted that the same assumption was made by Luikov (*15*). During the process of drying, the actual evaporative surface consists of a number of elements situated at varying positions relative to the surface depending on the moisture content. At the earliest stage, virtually all pores are filled at the surface and the effective position of the evaporative interface coincides with the physical surface of the porous medium. As the drying process proceeds, the removal of liquid from the pores produces liquid surface curvatures creating capillary forces which stress the entire specimen and result in the opening (emptying) of the largest surface pores. The new interfaces formed must of necessity be situated below the physical boundary so that the effective position of the evaporative plane must be below the surface of the porous medium. Continuation of the drying results in the opening of more surface pores, the creation of additional subsurface interfaces, and the progressive recession of the evaporative surface on an overall basis. The maintenance of relatively high drying rates over a considerable moisture content is due to the capillary forces which draw liquid to the evaporating sites at the expense of larger subsurface pores. Although axial variations in local evaporation rates tend to cause the mean evaporative surface position to be more submerged at the leading edge, there is a tendency for the liquid to offset this effect by flowing toward the regions of highest evaporation under the influence of the capillary pressure gradients. Experimental evidence (*1, 18*) suggests that there is relatively little variation in τ with x, except perhaps when τ and x are both very small. Accordingly, this assumption is not only convenient for purposes of analysis but might reasonably be expected to be a good first approximation of reality.

The steady state temperature distribution applies rigorously to those processes in which the evaporative interface is maintained at a fixed position. As applied to drying, the assumption will be reasonable only if the motion of the interface is sufficiently slow to permit a close approach to the steady state condition. Approximate calculations related to experimental studies (*18*) indicate that the temperature distribution above the evaporative interface should not deviate appreciably from the assumption. A more detailed analysis of this problem has been undertaken (*2*).

The treatment of the region above the evaporative interface as one-dimensional conduction excludes heat transfer by free convection or radiation. Kunii and Smith (*12*) have reported that radiation effects are negligible except for temperatures above 900°F. and for large particles. Similarly, free convection will be negligible for small pore spaces and small temperature differences.

The theory is essentially one-dimensional by the omission of axial conduction, although it predicts axial temperature gradients. Except for small τ and for the region near the leading edge when significant temperature gradients may exist, the effect would be expected to be small and would tend to reduce the axial variation of the evaporation rate.

The linear temperature distribution above the evaporation plane is a consequence not only of the condition of steady state and one-dimensional conduction but also of neglecting the influence of mass transfer on the temperature profile. The latter will apply only as $J_w(x)\ c_p/k_{\mathrm{eff}}$ approaches zero (*4*) and is approximated to a greater or lesser degree depending on $J_w(x)[c_p/k_{\mathrm{eff}}]\tau$. For low mass transfer and small values of τ, the linear temperature profile should be a good approximation.

Heat transfer by radiation between the porous medium and the surroundings has been neglected. In addition the lateral surfaces and the evaporative plane have been assumed to be adiabatic.

With regard to this last assumption, a portion of the energy transferred to the evaporative surface must be employed to meet the sensible energy requirements of the moist substrate. As a result, the surface energy balance, Equation (11), will be in error by an amount which becomes greater as the fraction of the total thermal energy which goes into vaporizing the liquid decreases. This fraction depends upon the heat of vaporization of the liquid and the heat capacity and thermal conductivity of the wet material below the plane of vaporization. During the drying process, the effective thermal conductivity in this region may be appreciably enhanced by an evaporation-diffusion-condensation cycle promoted by temperature gradients (*10*). Heat losses from the specimen to the surroundings increase the error accordingly.

The current steady state model can be extended to include sensible heat effects by letting η be the fraction of the thermal energy transferred toward the evaporative surface which is utilized to evaporate liquid, so that Equation (11) may be written as

$$\eta \frac{q_w(x)}{J_w(x)} = -\Delta H \tag{47}$$

Proceeding as in Part II, it can be shown (*18*) that the solution to the Volterra integral equation is identical with the previous solution, provided that the quantities $t_E(0)$ and t_{AS} are computed from modified forms of Equations (14) and (15), respectively, in which the right-hand sides are multiplied by the parameter η. These modified values of $t_E(0)$ and t_{AS} must then be used to compute the required parameters P and $q_w(0)$. In general, both P and $q_w(0)$ increase as η decreases, resulting in an increase in the driving force for heat transfer which tends somewhat to offset the effect of energy partition (*11*).

The assumed linear relationship between vapor pressure and temperature becomes a better approximation at low mass transfer rates because such rates are often achieved as a consequence of relatively small temperature differences between the air stream and the porous medium. Recently a detailed study of the consequences of this linearization has been undertaken (*11*). Preliminary results indicate that predicted evaporation rates are somewhat lower if an accurate vapor pressure relationship rather than a linear one is employed. In a series of computations for evaporation of water for psychrometric temperature differences $t_\infty - t_{WB}$ of from 34° to 166°F., differences in predicted evaporation rates of from 2 to 12% were obtained.

In view of the limitations of the theory described above, it would be desirable to compare predicted results with experimental data. Unfortunately no direct experimental measurements of steady state evaporation from submerged interfaces appear to have been reported. However, experimental drying data (*18*) support the present analysis in its major features. First, the theory predicts a rise in the temperature of the evaporative interface from the wet bulb to the pseudo wet-bulb temperature as the interface recedes into the depths of the porous medium.

Such a temperature variation has been reported by Adams (*1*), Luikov (*17*), and Nissan and co-workers (*19*). Second, the recession of the evaporative interface was deduced from drying data for a variety of liquids from beds of glass spheres (*18*). The resulting relation between moisture content and τ was similar for all systems studied. For moisture contents above 17%, relatively little recession was indicated in accord with the experimental observations of Shubin (*22*) and Newitt and co-workers (*20*). The latter noted that in the drying of randomly packed beds of spheres evaporation should result in the gradual retreat of surface menisci during the period in which liquid movement is controlled by capillary action. In the moisture content range of 8 to 17%, τ increased rapidly, agreeing with Adams' deductions, based on temperature and moisture distributions, that above an average moisture content of 13 to 16% there is negligible vaporization below a minute surface layer. Below these moisture contents, the bed rapidly passes into the pendular state in which liquid flow to the surface ceases and evaporation in situ followed by diffusion to the surface predominates.

In conclusion, the theory appears to provide a useful and valid approach to the analysis of drying data through the relation of the recession parameter τ to the moisture content. The predicted results, including the recession and the temperature rise, are in general accord with physical observations. In the more specific application to steady state evaporation from a submerged evaporative interface, the theoretical formulation is significantly less restricted.

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NOTATION

A, B = constants in linearized vapor pressure-temperature equation, Equation (12)
c = total molar concentration of vapor
c_n, d_n = coefficients for series expansions
C_n, D_n = ratios of gamma functions, as defined by Equations (30) and (24)
$\tilde{c}_p$ = molal heat capacity at constant pressure
c_T = constant in Equation (3)
D_v = mass diffusivity of gas mixture
D_{eff} = effective mass diffusivity of porous medium above plane of evaporation
$f(Pz)$ = local f function as defined by Equation (34)
$\overline{f}(Pz_L)$ = average $\overline{f}$ function as defined by Equation (36)
$\overline{g}(Pz_L)$ = average $\overline{g}$ function as defined by Equation (38)
$h_p(x)$ = local heat transfer coefficient as defined by Equation (42)
h_T, h_{TL} = heat transfer coefficient for a constant wall temperature; $h_{TL} = h_T(x)$ at $x = L$
$h(x, \xi)$ = heat transfer coefficient beyond a step in wall temperature at $x = \xi$
$J_w(x)$ = local wall molar flux of the diffusing component with respect to the molar average velocity
k = thermal conductivity of gas mixture
k_{eff} = effective thermal conductivity of porous medium above plane of evaporation
k_T, k_{TL} = mass transfer coefficient for a constant wall concentration; $k_{TL} = k_T(x)$ at $x = L$
$k(x, \xi)$ = mass transfer coefficient beyond a step in wall temperature at $x = \xi$
K = modified relative resistance parameter, $K = z/\sqrt{\pi}$
$K_1(x, \xi)$ = heat transfer kernel in Equation (5) as defined by Equation (7)
$K_2(x, \xi)$ = mass transfer kernel in Equation (8) analogous to $K_1(x, \xi)$
L = length of porous medium
m = exponent on local Reynolds number in Equation (3)
n = index for terms in series expansions, for example, Equations (20) and (28)
N_{Pr} = Prandtl number; $N_{Pr} = c_p\mu/k$
N_{Re_x} = local Reynolds number; $N_{Re_x} = U_\infty x/\nu$ for flow over a flat plate and $N_{Re_x} = v^* x/\nu$ for developed flow in a duct or channel
N_{Sc} = Schmidt number, $N_{Sc} = \nu/D_v$
p = exponent on Prandtl number in Equation (3)
P = parameter calculable from Equation (19) or (45)
$q_w(x)$ = local wall heat flux, that is, the local heat flux at the surface of the porous body
$q_w(0)$ = $q_w(x)$ at leading edge ($x = 0$)
$q_w(Av)$ = $q_w(x)$ averaged over the length of the porous body
$[q_w(Av)]_{Z \to \infty}$ = $q_w(Av)$ evaluated when evaporation takes place at the surface of the porous body
$t_E(x)$ = local temperature at evaporative surface
$t_E(0)$ = $t_E(x)$ evaluated at $x = 0$
$t_w(x)$ = local temperature at surface of porous body
t_{AS} = temperature at evaporative surface when evaporation occurs at the surface of the porous body
t_{WB} = wet-bulb temperature
t_∞ = free-stream temperature
u_{th} = mean longitudinal velocity across thermal boundary layer
U_∞ = free-stream velocity
v^* = friction velocity, $\sqrt{\tau_w/\rho}$; τ_w = wall shear stress
x = longitudinal distance measured from leading edge of porous medium
X = mole fraction of diffusing vapor component
y = transverse distance measured from surface of porous medium
z, z_L = relative resistance parameter for heat transfer, $z = k_{\text{eff}}/\tau h_T$; $z_L = z$ evaluated for $h_T = h_{TL}$

Greek Letters

α, δ = parameters in superposition factor, Equation (1)
Γ = gamma function
$\Delta H, \Delta H'$ = latent heat of vaporization, mole, and mass basis
ϵ = void fraction of porous medium
η = fraction of thermal energy incident on evaporative surface which is utilized to evaporate liquid
λ_1, λ_2 = constants as defined by Equations (7) and (9)
μ = viscosity
ν = kinematic viscosity, μ/ρ
ρ = density
τ = distance measured from evaporative surface to surface of porous body, that is, position of evaporative surface
ξ = position where step in wall temperature occurs, used as dummy variable in step-wall temperature superposition

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Heat Transfer and Binary Diffusion with Thermodynamic Coupling in Variable-Property Forced Convection on a Flat Plate

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The variable-property, laminar, boundary-layer equations, which describe simultaneous momentum, heat, and binary mass transfer with thermodynamic coupling,* are analyzed for air flows over a flat plate with the injection of foreign gases through the solid surface. A simplified general treatment of thermodynamic coupling is developed and applied to yield approximate but accurate expressions for evaluating heat transfer rates and adiabatic wall temperatures for injection of hydrogen, helium, and carbon dioxide. This method explains why the driving force based on $(T_w - T_{aw})$ can be used to correlate heat transfer results for situations where diffusion thermo is important. Furthermore, and perhaps most significant, the method provides a simple error estimate for the correlation obtained by using the adiabatic wall temperature.

It is shown that the injection of lightweight gases can significantly reduce viscous dissipation in flows over slender bodies and that diffusion thermo and dissipation effects can be of the same order of magnitude even for reasonably high-velocity flows. These effects are discussed in terms of convenient quantities, called σ functions.

Recent applications, particularly in the aerospace field, have intensified interest in finding suitable means for decreasing the rate of heat transfer from hot fluid layers to solid surfaces. This is primarily because thermally resistant materials for turbine blades, exhaust nozzles, combustion chamber walls, high-speed aircraft, reentry vehicles, etc, may be developed only within certain limits; beyond these a means must be found for maintaining tolerable surface temperatures by reducing heat transfer. Most of the methods proposed for this purpose involve the transfer of mass from or through the solid surface or mass transfer from a liquid film covering the solid surface. Thus a considerable body of literature on the problem of simultaneous heat and mass transfer has arisen and many of the most recent papers have been discussed by Sparrow (*11*) and by Zeh (*21*). These studies are all concerned primarily with heat

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* Thermodynamic coupling refers to the Soret (or thermal diffusion) effect, which is the flow of mass caused by a temperature gradient, and the Dufour (or diffusion thermo) effect, which is the flow of heat caused by a concentration gradient.